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On the stability of solutions of a system of differential equations

By

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Introduction. About the non-linear differential equation as well as about the linear one, various authors have discussed the boundedness, the convergence(1) of solutions and the existence of a periodic solution(2) which we have also discussed recently. Moreover we have obtained necessary and sufficient conditions for the boundedness of solutions of a certain type.(3) In these researches we have utilized the existence of a function of points which is characterised by the fact that it is non-increasing or non-decreasing along any solution of the given differential equation. Now by this idea, we will discuss the stability of solutions about which Liapounoff(4) and various mathematicians and physicists have discussed actively and they have obtained many remarkable results in connection with the above mentioned problems.

Now we consider a system of differential equations,

$$\frac{dy}{dx}=F(x, y), \tag{1}$$

where y denotes an n-dimensional vector and $F(x, y)$ is a given vector field which is defined and *continuous* in the domain

$$\Delta: 0\leqq x<\infty, \quad |y|\leqq R \quad (|y|=\sqrt{y_1^2+\cdots+y_n^2}).$$

Before the research of a solution, we assume at first that $y(x)\equiv 0$ is a solution of (1), so that $F(x, 0)$ is *identically* zero. And let $y=y(x; y_0, x_0)$ be any solution of (1) satisfying the initial condition $y=y_0$ when $x=x_0$.

Here we state the following definitions.(5)

a) The solution $y\equiv 0$ is said to be *stable* if, given any $\varepsilon>0$, there exists a $\delta>0$ such that if $|y_0|\leqq\delta$, then $|y(x; y_0, 0)|\leqq\varepsilon$ for all $x\geqq 0$.