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Kelvin principle and some inequalities in the theory of functions II

By

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In the previous paper[3] the author established, by means of the Kelvin minimum energy principle, several inequalities which may be reduced to the statements regarding the properties of harmonic functions with a vanishing normal derivative on some of boundary components of a given domain.

It is the object of this paper to deduce further inequalities of the same kind and supplement the previous one. For the sake of simplicity we shall use the same notations with that of the previous paper.

1. Generalization of Theorem I. In this section we shall generalize Theorem I to the case of a domain D which contains a number of mutually disjoint subdomains $D_1, D_2, \cdots, D_m$. We now have to introduce a number of singularity functions $S_\nu(z)$ $(\nu=1, \cdots, m)$, which are harmonic and single-valued in the closure of $D - D_\nu$. For the shorter formulation we restrict ourselves to the case of schlicht domains bounded by a finite number of analytic curves. Thus we obtain the following

THEOREM III. *Let D be a schlicht domain and let $D_1, \cdots, D_m$ be mutually disjoint subdomains of D. Let $S_\nu(z)$ $(\nu=1, \cdots, m)$ be the singularity functions defined above and let the functions $p_\nu(z)$ $(\nu=1, \cdots, m)$ be such that $\partial p_\nu/\partial n=0$ on the boundary C_ν of D_ν, and that $p_\nu(z)+S_\nu(z)$ is harmonic in D_ν. If $P(z)$ is a function which has a vanishing normal derivative on the boundary C of D and for which $P(z)+\sum_{\nu=1}^{m} S_\nu(z)$ is harmonic in D, then*

$$(1) \qquad \sum_{\nu=1}^{m} \int_{C_\nu} p_\nu \frac{\partial S}{\partial n} ds \geqq \int_C P \frac{\partial S}{\partial n} ds$$