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Note on the continuation of harmonic and analytic functions

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1. In the present paper we shall state some notes concerned with the following problem for which P. J. Myrberg has found a wonderful result[1]:

Let the notations $HB(AB)$, $HD(AD)$ denote respectively the classes of bounded harmonic (analytic) functions and of harmonic (analytic) functions with bounded Dirichlet integral. Let R be an arbitrary Riemann surface and E be a closed subset of R. Then what conditions are necessary and sufficient, in order that E should be *removable* for each family defined on $R-E$ (i.e., it would be possible to continuate without singularities all the functions belonged to the class harmonically or analytically onto E)?

2. The case of HB and HD

Lemma.[2] *Let R be an arbitrary Riemann surface of hyperbolic type. Let $g(P, P_0)$ be the Green's function on R with a pole P_0 and let U be an arbitrary neighbourhood which contains the pole P_0. Then the Dirichlet integral $D_{R-U}[g]$ of g taken over $R-U$ is finite, especially*

$$D_{R-U}[g]=\int_{\partial U} g d\bar{g}$$

if the boundary[3] *∂U of U is analytic, where, in general, the barred letter stands for the conjugate harmonic function.*

Proof. It suffices to assume that ∂U is analytic. Consider the exhaustion of R

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1) P. J. Myrberg: Über die analytische Fortsetzung von beschränkten Funktionen. Ann. Acad. Sci. Fenn. Ser. A, I. 58 (1949).

2) Cf. Nevanlinna: Uniformisierung. 1953.

3) In the following ∂A denotes the boundary of A.