MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXIX, Mathematics No. 1, 1955.

On the dimension of local rings

By

Mieo NISHI

(Received June 1, 1954)

Let A be a local ring in Krull's sense [3][1] and $\mathfrak{p}$ any prime ideal of A. Then in general the sum of the rank and the dimension of $\mathfrak{p}$ is at most equal to the dimension of A. But there are several types of local rings in which, for any prime ideal, the sum of the rank and the dimension is exactly equal to the dimension of the ring itself; that is, for regular local rings this was proved by Krull [3], for geometrical local rings by C. Chevalley [1] and for complete local domains by I. S. Cohen [2]. In this short paper we shall give the most general result which includes each case above-mentioned.

Let A be a local ring and $\hat{A}$ its completion; and let n be the dimension of A. Then as is well known $\dim \hat{A} = \dim A = n$. For any prime ideal $\mathfrak{p}$ of A we have

$$\dim A/\mathfrak{p} + \dim A_{\mathfrak{p}} \leq n^{2)},$$

as is mentioned above. But now we shall prove that if $\dim \hat{A}/\hat{\mathfrak{p}}_i^{(0)}$ is equal to n for every minimal prime divisor $\hat{\mathfrak{p}}_i^{(0)}$ of (0) in $\hat{A}$, then the equality holds.

First we assume that A is an integral domain. Let us put $\dim A_{\mathfrak{p}} = r$, then there exists a set of elements $\{a_i \in \mathfrak{p}\,;\, i=1, 2, \cdots, r\}$ such that $\mathfrak{p}$ is a minimal prime divisor of $(a_1, a_2, \cdots, a_r)A$. For,

---

1) Numbers in brackets refer to the bibliography at the end of this paper.

2) Throughout this paper $A_{\mathfrak{p}}$ is meant by the ring of quotients in Chevally's sense. That is, let S be the complementary set of the prime ideal $\mathfrak{p}$ in A, then the set $N = \{a \in A\,;\, ab = 0 \text{ for some } b \in S\}$ is an ideal of A and there exists a natural homomorphism ϕ of A into A/N. As is easily seen, $\phi(S)$ has no zero divisors in $\phi(A)$ and hence we can define the ring of quotients in Grell's sense with respect to $\phi(A)$. This is denoted by $A_{\mathfrak{p}}$.

Cf. Chevalley: On the notion of the ring of quotients of a prime ideal, Bull. Amer. Math. Soc., Vol, 50 (1944).