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On the hypersurface sections of algebraic varieties embedded in a projective space

By

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The problem to decide whether the intersection-product of two varieties is irreducible or not seems to us very important in algebraic geometry. Recently Prof. Chevalley proposed to us a problem of this character. (The final form of the problem is stated in Theorem 2 in this paper.) We heard that Chevalley has a proof already. But the proof in this paper is simple and quite different from his principle. We wish to express our hearty thanks to Prof. Akizuki for his kind encouragement.

Let $\boldsymbol{L}^N$ be a projective space of dimension N. Then as is well known every positive cycle $\boldsymbol{X}$ in $\boldsymbol{L}^N$ can be represented as a point of a suitable projective space by the coefficients of the associated form of $\boldsymbol{X}$.[1] We shall call it the Chow point of $\boldsymbol{X}$ and denote by $C(\boldsymbol{X})$. In particular the totality of Chow points of the hypersurfaces of order m in $\boldsymbol{L}^N$ constitutes a projective space $\boldsymbol{L}'^{l(N,m)}$ of dimension $l(N, m)=\binom{N+m}{N}-1$.

LEMMA 1. *Let $\boldsymbol{H}_1$ and $\boldsymbol{H}_m$ be a hyperplane and a hypersurface in $\boldsymbol{L}^N$ such that* $\dim_k C(\boldsymbol{H}_m)=l(N, m)-g$ *and that the intersection-product $\boldsymbol{H}_1\cdot\boldsymbol{H}_m$ is defined, then we have* $\dim_{k(C(H_1))} C(\boldsymbol{H}_1\cdot\boldsymbol{H}_m)\geq l(N, m)-g-N$, *where k is any field over which $\boldsymbol{L}^N$ is defined and g any posive integer.*

PROOF. As is easily shown we have

$$\dim_{k(C(H_1))} C(\boldsymbol{H}_m)\geq l(N, m)-g-N\,.$$

Hence there exist[2] on $\boldsymbol{H}_m$ at least $l(N, m)-g-N$ independent

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1) Cf. B. L. van der Waerden, "Einführung in die algebraische Geometrie." Julius Springer in Berlin, 1939.

2) Cf. Y. Nakai, "Notes on Chow points of algebraic varieties." Mem. Coll. Sci., Univ. of Kyoto, vol. XXVIII, 1953.