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Derivation and cohomology in simple and other rings. II

(A remark on the Kronecker product $A \times_C A$)

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In our first paper I[1] we proved first that if A is a simple ring (having unit element 1 and satisfying minimum condition) and if C is a weakly normal simple subring of A (which contains 1 and over which A is assumed to be finite for the sake of simplicity), then the Kronecker product (or, direct product, as we called it in I) $A \times_C A$ over C is completely reducible as A-C-double-module, under ordinary operation.[2] This we proved indeed by combining the following two facts, which were proved either in I or in a former paper of the writer: Under the same assumption, 1) the A-C-module A is completely reducible; 2) $A \times_C A$ is A-two-sided completely reducible and is a direct sum of minimal A-double-submodules which are A-left-semilinearly and A-right-linearly isomorphic to A. Thus arises our interest in investigating the relationship between the A-two-sided complete reducibility of $A \times_C A$ and the A-C-complete reducibility of A itself, where A is a ring with unit element 1 and C is a subring of A which contains 1. A typical case, where we have the latter but not the former, is the case of a field C and a non-separable semisimple algebra A over C. It is also clear that the former does not imply the latter in general. For instance, let A be the complete matric ring $\varepsilon_{11}\Omega + \varepsilon_{12}\Omega + \varepsilon_{21}\Omega + \varepsilon_{22}\Omega$ over a field Ω and C be its subring $\Omega + \varepsilon_{21}\Omega$; observe that A has even an (independent) two-sided basis over C,

1) Duke Math. J. 19 (1952), 51-63.

2) We proved the same also under Hochschild's cohomological operation. Further we considered Kronecker products $A \times_C A \times_C \cdots \times_C A$ with more factors than 2, and proved their A-B-complete reducibility, where B is any (necessarily weakly normal) simple subring of A which contains C.