MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXIX, Mathematics No. 3, 1955.

# On the derived normal rings of Noetherian integral domains

By

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(Received October 16, 1955)

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It was communicated to the writer that Y. Mori [4, II][1] proved that the derived normal ring of a Noetherian integral domain of rank 2 is also Noetherian.[2] In the present paper we want to give a new proof of the result. In the way, we shall show a detailed and a little clearer proof of the result due to Y. Mori [4] that the derived normal ring of a Noetherian integral domain is a Krull ring.[3]

Terminology and results stated in Nagata [7] will be used freely. Further, some basic results on local rings (see, for example [8, § 1]) will be used freely.

## § 1. Krull-Akizuki's theorem.

PROPOSITION 1 (Krull-Akizuki's theorem).[4] *Let $\mathfrak{o}$ be a Noetherian integral domain of rank* 1 *and let $K$ be the field of quotients of $\mathfrak{o}$. Let $L$ be a finite algebraic extension of $K$. If $\mathfrak{o}'$ is a subring of $L$ containing $\mathfrak{o}$, then for every ideal $\mathfrak{a}'(\neq 0)$ of $\mathfrak{o}'$, $\mathfrak{o}'/\mathfrak{a}'$ is a finite $\mathfrak{o}/(\mathfrak{a}' \cap \mathfrak{o})$-module. Consequently, $\mathfrak{o}'$ is Noetherian.*

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1) The result was anounced by him at the Autumn meeting of the Mathematical Society of Japan in 1953.

2) As was shown by Nagata [6], i) there exists an example of Noetherian integral domain $\mathfrak{o}$ of rank 2 which has an integral extension $\mathfrak{o}'$ contained in the derived normal ring of $\mathfrak{o}$ such that $\mathfrak{o}'$ is not Noetherian and ii) there exists an example of Noetherian integral domain of rank 3 such that the derived normal ring is not Noetherian. On the other hand, as is well known as Krull-Akizuki's theorem, when $\mathfrak{o}$ is a Noetherian integral domain of rank 1, then every almost finite integral extension of $\mathfrak{o}$ is Noetherian (see § 1).

3) For the definition, see § 2.

4) The writer owes the present formulation of the theorem to Cohen [2].