

Note on the boundedness and the ultimate boundedness of solutions of $x' = F(t, x)$

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In order to obtain existence theorems of periodic solutions of the non-linear differential equation of the second order, Reuter [5]* and various authors have discussed the boundedness of solutions. The present author has also researched conditions for the boundedness or the ultimate boundedness of solutions for the purpose of using Massera's theorem (Theorem 2 in [4]) in the discussion of the existence of a periodic solution ([6], [7] and [10]). And utilizing the properties of solutions, the author has obtained necessary and sufficient conditions for the boundedness of solutions ([8] and [10]). A certain function which appears in these conditions resembles that of Liapounoff's research [2] for the stability of the solution.

The author thinks that the stability and the asymptotic stability correspond to the boundedness and the ultimate boundedness respectively and *in certain sense* they are of the same concepts respectively. Of course, with regard to the independent variable, both are of the problems "*in the large*", but with regard to unknown functions, the former is of the problem "*in the small*", while the other is of the problem "*in the large*". Now we will discuss the boundedness and the ultimate boundedness. Massera [3] and several authors have discussed problems reciprocal to Liapounoff's condition for the stability. We will see that we obtain some results analogous to those of their researches. The author has also obtained necessary and sufficient conditions for the stability ([9]) and of course we can obtain results for boundednesses analogous to them.

Now we consider a system of differential equations,

* Numbers in [] refer to the bibliography at the end of the paper.