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Some remarks on invariant forms of a sphere bundle with connexion

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Let us consider an $(n-1)$-sphere bundle $\mathfrak{B}^{n-1}(M, S^{n-1}, O_n^+)$ over a differentiable manifold M with the proper orthogonal group O_n^+ of degree n. Let $\mathfrak{B}^q(M, Y^q, O_n^+)$ $(0 \leqq q \leqq n-1)$ denote the associated bundle of $\mathfrak{B}^{n-1}$ with the Stiefel manifold $Y^q = O_n^+/O_q^+$ as fibre. Then, the associated principal bundle $\mathfrak{B}^0(M, O_n^+)$ can be also regared as principal bundles $\mathfrak{B}^0(\mathfrak{B}^q, O_n^+)$ over $\mathfrak{B}^q$ with groups O_q^+. From a connexion defined on $\mathfrak{B}^0(M, O_n^+)$, we can induce naturally connexions on $\mathfrak{B}^0(\mathfrak{B}^q, O_q^+)$. In the present paper we show that by employing these induced connexions, the formulas in our preceding papers[1)] and their generalizations can be expressed in a simple manner

§ **1.** Let V be an r-dimensional vector space over the real number field R. Its exterior algebra Λ is a graded ring whose homogeneous elements of degree k constitute the space $\Lambda^k (0 \leqq k \leqq n)$ of all exteior k-vectors; in particular $\Lambda^0 = R$ and $\Lambda^1 = V$. Let M be a differentiable manifoid. For the sake of simplicity, we understand that the term "differentiable" means always the differentiability of suitable class. We denote by $T(M)$ and $T_x(M)$ the tangent vector bundle over M and the tangent vecter space of M at $x \in M$ respectively. From any differentiable mapping $\varphi: M \rightarrow M'$, we can induce a linear map $T_x(M) \rightarrow T_{\varphi(x)}(M')$ which we shall denote by φ^*. We consider a p-form θ with values in Λ^k: to any set of vectors $t_1, \cdots, t_p \in T_x(M)$ $x \in M$, is assigned an element $\theta(t_1, \cdots, t_p) \in \Lambda^k$

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1) S. Takizawa; *On the Stiefel characteristic classes of a Riemannian manifold.*
————; *On the primary difference of two frame functions in a Riemannian manifold.*
T. Yagyu; *On the Whitney characteristic classes of the normal bundle.* These memoirs, 28, No. 1 (1953)