

Addition and corrections to my paper "A treatise on the 14-th problem of Hilbert"

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Concerning the 14-th problem of Hilbert, Zariski [3] conjectured the following :

Conjecture of Zariski. Let D be a positive divisor on a normal projective variety V defined over a field k and let $R[D]$ be the set of functions f on V defined over k such that $(f) + nD > 0$ for some natural number n . Then $R[D]$ will be an affine ring over k .

He proved there that if the answer of this conjecture is affirmative, then the answer of the following problem is affirmative :

The generalized 14-th problem of Hilbert : Let \mathfrak{o} be a normal affine ring over a field k and let L' be a function field contained in the function field of \mathfrak{o} . Is then $\mathfrak{o} \cap L'$ an affine ring?

In the present paper, we shall show at first that the generalized 14-th problem of Hilbert is equivalent to the conjecture of Zariski and then we shall give some corrections to my paper [2].

§ 1. The proof of the equivalence.

Since Zariski [3] proved that the affirmative answer of the conjecture of Zariski implies the affirmative answer of the generalized 14-th problem of Hilbert, we have only to prove the converse. The writer proved in [2] that the generalized 14-th problem is equivalent to

Problem A. Let \mathfrak{a} be an ideal of a normal affine ring \mathfrak{o} over a field k . Is then the \mathfrak{a} -transform of \mathfrak{o} an affine ring?

Therefore we have only to prove that :

The affirmative answer of Problem A implies the affirmative answer of the conjecture of Zariski.

Now we shall use the notations as in the conjecture of Zariski. Let L be the field of quotients of $R[D]$ and let \mathfrak{o} be a normal