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On the arithmetic genera and the effective genera of algebraic curves

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In the study of algebraic curves in a fixed projective space (or on a non-singular surface) the arithmetic genera and the effective genera of curves are taken as their most important numerical characterizations.

In this paper we want to pick up some of their fundamental properties and to discuss on them without any restriction on the characteristic of the universal domain.

Throughout this paper we shall fix a projective N-space $\boldsymbol{P}^N$ ($N \geqq 2$) and say briefly *a curve* in-stead of a positive 1-dimensional $\boldsymbol{P}^N$-cycle without multiple components. As usual, for a given curve in $\boldsymbol{P}^N$, we write down $1-p_a$ for the constant term of the Hilbert characteristic function of the curve and we call the integer p_a the *arithmetic genus* of the curve; for an irreducible curve the *effective genus* means the arithmetic genus of a non-singular[1] curve which is birationally equivalent to it, and in general for a reducible curve it is defined by the sum of those of the absolutely irreducible components of the curve. Part (I) will be devoted to preliminary definitions and studies on intersection multiplicity and order of singularity from the local view-point; the intersection multiplicity defined in this paper is a generalization of the usual one which we shall need in Part (II) in order to state a generalized modular property of the arithmetic genera of curves. Also in Part (II) we shall prove a formula which expresses the difference of the arithmetic and the effective genera of a curve in terms of the orders of singularity at singular points, which includes the classical genus formula of a plane curve as a special case.

Finally in Part (III), we shall study the order of newly out-