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On the algebraic theory of sheets of an algebraic variety

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The motivation of the present note is stated at the begining of [1]. In order to determine the dimension of the algebraic subsystem of curves with prescribed number of nodes in a linear system on a surface, Severi used the " method of analytic branches " successfully. ([9] Anhang F, [10] p. 85 and p. 167.) The preceding two papers [1], [8] have shown how one can avoid explicit use of the analytic method, but it seems nevertheless to be of some interest to translate the method of analytic branches (or sheets) into the language of local algebra, which we will do in this note.

In § 1 we shall define the *decomposition number* dn($\mathfrak{p}$) of a prime ideal $\mathfrak{p}$ of a ring $\mathfrak{o}$, which is an algebraic translation of the geometric notion of the number of sheets of a variety at a point. From a property of the decomposition number we deduce the following proposition: *Let V be an r-dimensional algebraic variety. Then those points of V which decompose into d or more points on the derived normal variety $\widetilde{V}$ of V, together with their specializations, make up an algebraic subset $\mathfrak{B}_d$ of V. If V is embedded in an $(r+1)$-dimensional non-singular variety U, then each component of $\mathfrak{B}_d$ has dimension* $\geq r+1-d$. Here one may replace " $P(\in V)$ decomposes into d points on $\widetilde{V}$" by " V has d sheets at P" by virtue of Zariski's theorem of analytic normality, as is well known.

In § 2 we shall apply the above proposition to the geometric problem treated in [1] and [8], and shall show how well does Severi's reasoning work also in the abstract case.

In § 3 we shall show, by means of the theory of Henselian rings, that a component of the intersection of several sheets of an