

A property of an ample linear system on a non-singular variety

By

Yoshikazu NAKAI

(Received January 10, 1957)

We shall treat here the same subject as is stated in the preceding paper¹⁾ using the dual map into the Grassmann variety. The contents of this paper are almost as the same as the contents of § 3 of my paper "On the characteristic linear systems of algebraic families" (will appear in Illinois' Journal), but I would like to present here again as a memory of Prof. Zariski following the advice of Prof. Akizuki. Before to state the complete form of the final result we must introduce some auxiliary notions.

Let V be an irreducible variety and E be an ample linear system of divisors on V without fixed component. Then E defines an everywhere biregular birational transformation of V onto a projective variety V_E . Let $n = \dim E$, and k a common field of definition for V and E . Then the variety V_E is defined over k , and belongs to a projective space L^n (i. e. not contained in any hyperplane of L). Let P, \bar{P} be the corresponding generic points of V, V_E over k and $T_{\bar{P}}$ the tangent linear variety to V_E at \bar{P} . Then the Plücker coordinates $c(T_{\bar{P}})$ is rational over $k(P)$, and the point $c(T_{\bar{P}})$ has a locus V_E^* over k . We shall call this variety *the dual variety of V with respect to the linear system E* , and the map φ_E of V onto V_E^* defined over k by $\varphi_E(P) = c(T_{\bar{P}})$ will be called the *dual map* of V onto V_E^* . The map φ_E is defined at every simple point of V .

Now our theorem is as follows.

Theorem 1. *Let E be an ample linear system on a non-singular variety V defined over k and assume that the dual map φ_E of V*

1) Y. Akizuki and H. Matsumura, On the dimensions of algebraic system of curves with nodes on a surface, in the same number of this Memoirs.