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On the dimension of algebraic system of curves with nodes on a non-singular surface

By

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In this note we shall give an algebraic proof of the following

Theorem. *Let E be the linear system cut out on a non-singular surface by the hypersurfaces of order m. Then those curves of E which carry d or more nodes and no other kinds of singularities, together with their specializations, form an algebraic subsystem of E, and (if it is not vacuous) each of its components has dimension* $\geq \dim E-d$.

This theorem, as Severi showed already, plays an important rôle for a proof of the completeness of characteristic linear series in the classical case. The problem on the completeness of characteristic series in abstract case was taken up in the lectures of Zariski on minimal model of algebraic surfaces held in our department (October 1956),[1] and he conjectured that the problem would be true when the geometric genus is zero. Nakai proved at first the linearity of the characteristic series, but the question of the completeness remained unsolved until Zariski left here. The only missing point was the above theorem. Then we discussed on the problem in our seminar, thus we arrived at several proofs. So we wish to publish in the following papers, to which this paper also belongs, different proofs by several authors as a dear memory of Zariski and his lectures in Kyoto.

The principle of this first paper is very simple and elementary. So before stating the exact proof we shall explain the main idea by taking up the case of plane curves.

Let $\mathfrak{F}$ be an irreducible component of the algebraic system of

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1) The contents of his lectures will be published soon as a series of his papers on minimal model and rationality of algebraic surfaces.