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The differential geometry of spaces with analytic distances

By

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We consider here an analytic n-manifold V with a distance function $d(p, q)$ defined for all points p, q of V, satisfying the following two axioms:

1. $d(p, q)=0$ for $p=q$,
2. $d(p, q)=d(q, p)$;

at present we do not assume the so-called triangle axiom.

Further we suppose that there exists a coordinate neighborhood U containing any pair of points p, q and that the square of $d(p, q)$ is analytically expressed by the coordinates (x) and (y) of p and q respectively. Such a manifold V will be called the space with analytic distance and denoted by S^n. The function $g(x, y)$ defined by

$$g(x, y)=-\frac{1}{2}[d(x, y)]^2$$

will be called *fundamental function* of the space S^n and we shall investigate the properties of S^n from the standpoint of the differential geometry by means of the analyticity of the fundamental function.

The space with analytic distance was formerly investigated by one of my senior *Tsutomu Ôtake*, who died about ten years ago without publishing his note. In this paper we shall introduce and develope his discussions.

§ 1. Tensors in the space S^n.

It is natural by means of the geometrical meaning that the