MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXX, Mathematics No. 2, 1957.

On the induced connexions

By

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(Received October 1, 1956)

Dedicated to Prof. Jôyô Kanitani on His Sixty-third Birthday

Introduction

In the theory of connexions on differentiable principal bundles, the term " Induced Connexion " may be used in several different significances. What we shall introduce in the present paper is, however, one of the essential generalizations of the classical process of induction in the theory of submanifolds. It can be shown that a connexion is naturally induced on a certain principal bundle, whose bundle space and base space are both associated bundles of a given principal bundle with connexion.

In § 2, we give the definition and expositions of induced connexion, and derive generalizations of the equations of Gauss-Codazzi-Ricci. The structure of induced connexion appears not only in the theory of submanifolds, but also in the theories of various categories, for instance, canonical connexions on universal bundles, the Stiefel-Whitney characteristic classes, and reductive Cartan connexions. In the last four sections, we describe their applications.

§ 1. Survey of tensor calculus

We denote by $T(M)$ the tangent vector bundle over any C^∞-manifold M, and by $T_x(M)$ the tangent vector space at $x \in M$. Let $T^k(M)$ denote the space of all ordered sets $(t_1, \cdots, t_k)$ of tangent vectors such that $t_1, \cdots, t_k \in T_x(M)$, $x \in M$. Then $T^k(M)$ can be regarded as an associated bundle of $T(M)$. Let V be a vector space over the field of real numbers R. A *V-valued k-form* on M is, by definition, a C^∞-map

$$\theta : T^k(M) \rightarrow V,$$