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Appendix to the paper " Note on the boundedness and the ultimate boundedness "

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In the foregoing paper [2] we have discussed the boundedness and the ultimate boundedness of solutions of a system of differential equations. Now we consider a system of differential equations,

$$\frac{dx}{dt}=F(t, x), \tag{1}$$

where x denotes an n-dimensional vector and $F(t, x)$ is a given vector field which is defined and continuous in the domain

$$\Delta: \quad 0 \leqq t < \infty, \quad |x| < \infty.$$

$|x|$ represents *the sum of the squares* of its components. And let

$$x=x(t\,;\, x_0, t_0)$$

be a solution through the initial point (t_0, x_0). Except otherwise stated, we adopt the symbols and the promises in [2].

At first, we will discuss the boundedness of solutions under perturbations. Corresponding to the differential equation (1), we consider an equation

$$\frac{dx}{dt}=F(t, x)+H(t, x), \tag{2}$$

where $H(t, x)$ is a continuous vector field defined in Δ. Here we give a definition for the boundedness which corresponds to the stability under constantly acting perturbations.

Definition. The solutions of (1) are said to be *totally bounded* (or *bounded under constantly acting perturbations*), if for any $\alpha>0$, there exist two positive numbers β and γ such that, if $|x_0| \leqq \alpha$, then we have $|x(t; x_0, t_0)|<\beta$ for any $t \geqq t_0$ (t_0, arbitrary), where $x=x(t; x_0, t_0)$ is the solution of the equation (2) in which we have