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Note on the generator of $\pi_7(SO(n))$

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The 7th homotopy group $\pi_7(SO(n))$ of the group $SO(n)$ of the rotations in the euclidean n-space is determined by Serre [5] without details. Let

$$\sigma : S^7 \to SO(8) \text{ and } \rho : S^7 \to SO(7) \subset SO(8)$$

be mappings defined by the formulas

$$\sigma(x)(y) = xy \text{ and } \rho(x)(y) = xy\bar{x} \text{ for } x, y \in S^7,$$

where the multiplication in S^7 is that of the Cayley numbers.

Denote by

$$\sigma_n \in \pi_7(SO(n)),\ n \geqq 8 \text{ and } \rho_n \in \pi_7(SO(n)),\ n \geqq 7$$

the classes represented by σ and ρ respectively, regarding $SO(8)$ as a subgroup of $SO(n)$, $n \geqq 8$ in the natural sense. About the element ρ_7, we have the knowledge of the result [8]:

$$p_* \rho_7 \neq 0$$

under the (projection) homomorphism $p_* : \pi_7(SO(7)) \to \pi_7(S^6) \approx Z_2$. From this we can prove that "*ρ_7 is not divisible by 2*". Furthermore, we shall prove

Theorem. i) *$\pi_7(SO(7))$ is a free cyclic group generated by ρ_7.* ii) *$\pi_7(SO(n))$, $n \geqq 9$, is a free cyclic group generated by σ_n.*

As a corollary we have $\pi_7(SO(8)) \approx Z + Z = \{\sigma_8\} + \{\rho_8\}$.

The proof of the theorem is mainly devoted to the following simple lemma and results on $\pi_6(S^3)$.

$SO(7)$ is the set of all $\alpha \in SO(8)$ such that α fixes the unit. Spin(7) is the set of all $\tilde{\alpha} \in SO(8)$ such that for some $\alpha \in SO(7)$ the relation

$$\alpha(x)\tilde{\alpha}(y) = \tilde{\alpha}(xy)$$