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On the necessary and sufficient condition for the uniform boundedness of solutions of $x'=F(t, x)$

By

Taro YOSHIZAWA

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The stability of the solution of a system of differential equations has been discussed by Liapounoff [2]* and various authors. Massera has discussed the Liapounoff's function and the relations between various stabilities and has obtained necessary and sufficient conditions for the asymptotic stability of the solution ([3] and [4]). The present author has researched the boundedness of solutions which is the concept corresponding to the stability. And we have obtained sufficient conditions for the boundedness or the ultimate boundedness of solutions for the purpose of using Massera's theorem in the discussion of the existence of a periodic solution of the non-linear differential equation of the second order ([5] and [6]). Moreover it has been shown that those conditions are also necessary conditions for some equations ([9] and [10]). In the papers [7] and [8] we have obtained some necessary and sufficient conditions for the uniform boundedness or the uniform stability, but they are not satisfactory in practical cases, because they are the conditions which are satisfied for general equations. Then we will obtain necessary conditions for the uniform boundedness or the uniform stability.

Now we consider a system of differential equations,

$$\frac{dx}{dt}=F(t, x), \tag{1}$$

where x denotes an n-dimensional vector and $F(t, x)$ is a given

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* Numbers in [ ] refer to the bibliography at the end of the paper.