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An example of a normal local ring which is analytically reducible

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Previously the writer [2] gave an example of a normal local ring which is analytically ramified. In that example, the zero ideal of its completion is a primary ideal. In the present note, we shall show a normal local ring such that the zero ideal of its completion is not primary, which gives a counter example to a problem of the writer [3].

We shall remark here that our example is a finite separable integral extension of a regular local ring of rank 2 which contains a field k whose characteristic may be zero: we shall construct an example under the assumption that the characteristic of k is different from 2. It should be noted that even in the case of characteristic 2, a similar example can be given easily by a slight modification of our example.

(1) Construction of the example.

Let k be a field of characteristic not equal to 2. Let $w=\sum a_i x^i$ ($a_0=0$, $a_i \in k$) be an element of the formal power series ring $k\{x\}$ over k such that w is transcendental over $k(x)$.

Now, let x, y, z be algebraically independent elements over k and set $z_1=z$, $z_{i+1}=[z-(y+\sum_{j<i} a_j x^j)^2]/x^i$. Set $\mathfrak{r}=k[x, y, z_1, z_2, \cdots]_{(x, y, z_1, z_2, \cdots)}$. Then $\mathfrak{o}=\mathfrak{r}[W]/(W^2-z)$ is the required example.

(2) Properties of the ring $\mathfrak{r}$.

Since w is transcendental over $k(x)$, we can identify z with the element $(y+w)^2$ in the power series ring $k\{x, y\}$. Then as is easily seen, every z_i is identified with an element of $k\{x, y\}$ whose