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Relative Riemannian Geometry

I. On the affine connection

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Preliminaries

In a previous paper [1] we treated spaces with an analytic distance, in which an analytic distance-function $d(x, y)$ was given and a fundamental tensor $g_{i(j)}(x, y)$ was introduced by means of the distance such that

$$g_{i(j)}=\frac{\partial^2 g(x,\ y)}{\partial x^i\,\partial y^{(j)}},\qquad g=-\frac{1}{2}\,(d(x,\ y))^2\,.$$

From this we got the curvature tensor and some of the geometric notions. But it is clear that these can be derived from any function which is not necessarily the function as above given, and hence we can not expect many geometric notions enough to discuss the properties of the space. On the other hand we are under the consideration of the geometric interpretation of a system of integral equations

$$v^i(x)=u^i(x)-\int k^i_{j'}(x,\ y)u^{j'}(y)dy^{1'}\cdots dy^{n'},$$

where the u and v are vectors and the kernel $k^i_{j'}(x, y)$ is the tensor with respect to a pair of points (x, y). Further we should assume from the geometric stand-point that the kernel is of weight one with respect to (y). Thus we meet also with a notion of a tensor with respect to a pair of points.

From these view-points we shall introduce in this paper a notion of *a relative affine connection of a pair of manifolds* (M, N). The connection in M is determined in relation to so-called *observ-*