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On exact sequences in Steenrod algebra mod. 2

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A Steenrod algebra A^* will mean a stable Eilenberg-MacLane cohomology group $A^*(Z_2, Z_2) = \lim H^*(Z_2, n; Z_2)$ in which the multiplication is defined by the composition of the squaring operations Sq^t. The formula $\varphi_a(b) = ba$ associates for each element a of A^* an (additive) homomorphism $\varphi_a: A^* \to A^*$. We write $\varphi_a = \varphi_t$ if $a = Sq^t$, then $A^*(Z, Z_2) = A^*/\varphi_1 A^*$. We shall give an elementary proof of the following

Theorem I. *The following two sequences of homomorphisms are exact.*

$$\begin{array}{ccc} A^* & \xrightarrow{\varphi_2} & A^* \\ \uparrow \varphi_3 & & \downarrow \varphi_2 \\ A^*/\varphi_1 A^* & \xleftarrow{\varphi_5} & A^*/\varphi_1 A^* \end{array},$$

$$A^*/\varphi_1 A^* \xrightarrow{\varphi_3} A^*/\varphi_1 A^* \xrightarrow{\varphi_3} A^*/\varphi_1 A^* .$$

Several exact sequences are known experimentally for lower dimensions. For example, it seems that the sequence

$$A^* \xrightarrow{\varphi_{2^r}} A^*/(\sum_{i=0}^{r-2} \varphi_{2^i} A^*) \xrightarrow{\varphi_{2^r}} A^*/(\sum_{i=0}^{r-1} \varphi_{2^i} A^*)$$

is exact. More generally we propose

Problem. *Let* $a, b_1, \cdots, b_r \in A^*$. *Is the kernel of* $\varphi_a: A^* \to A^*/(\sum_{i=1}^{r} \varphi_{b_i} A^*)$ *finitely generated (as a left ideal)?*

In place of φ_a, take a homomorphism φ_a^* defined by the formula $\varphi_a^*(b) = ab$, then the exactness of analogous sequences is proved by T. Yamanoshita and A. Negishi (*cf.* [5]).