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p-primary components of homotopy groups

I. Exact sequences in Steenrod algebra

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The structure of the Steenrod algebra $\mathscr{S}^*$ mod p [1] gives important tools for the calculation of the homotopy groups. In this section, the exactness of the several $\mathscr{S}^*$-homomorphisms is studied, and it will be applied to prove the triviality of mod p Hopf invariant in the next section and also to verify the homotopy groups in those sections which follow further.

§ Notations.

Throughout this paper, p denotes an odd prime and $\mathscr{S}^*$ denotes the Steenrod algebra mod p [1] [3]. $\mathscr{S}^*$ is a graded Z_p-algebra $\sum_i \mathscr{S}^i$ which is generated multiplicatively by the Bockstein operator $\Delta \in \mathscr{S}^1$ and Steenrod's reduced powers $\mathscr{P}^t \in \mathscr{S}^{2t(p-1)}$, $t=0, 1, 2, \cdots$.

For the simplicity of the descriptions, we shall use the following notations.

(1.1) $\mathscr{P}(\Delta^{\varepsilon_0}, r_1, \Delta^{\varepsilon_1}, r_2, \cdots, r_n, \Delta^{\varepsilon_n}) = \Delta^{\varepsilon_0}\mathscr{P}^{r_1}\Delta^{\varepsilon_1}\mathscr{P}^{r_2}\cdots\mathscr{P}^{r_n}\Delta^{\varepsilon_n}$,

where ε_i and r_i are non-negative integers. From the relation

$$\Delta^2 = \Delta\Delta = 0,$$

the monomial (1.1) vanishes if one of $\varepsilon_i \geqq 2$. If $\varepsilon_i = 0$, we may omit Δ^{ε_i} in (1.1) since Δ^0 means the identity. If $\varepsilon_i = 1$, we write Δ^{ε_i} by Δ. Also if $r_i = 0$, then we may replace "$\Delta^{\varepsilon_{i-1}}, r_i, \Delta^{\varepsilon_i}$" and "$\Delta^{\varepsilon_{i-1}}\mathscr{P}^{r_i}\Delta^{\varepsilon_i}$" by "$\Delta^{\varepsilon_{i-1}+\varepsilon_i}$" since $\mathscr{P}^0$ is the identity.

A monomial (1.1) is said to be *admissible* if ε_i are 0 or 1, $r_n > 0$ and if $r_i \geqq pr_{i+1} + \varepsilon_i$ for $i = 1, 2, \cdots, n-1$. Then the admis-