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p-primary components of homotopy groups

III. Stable groups of the sphere

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Denote by $\pi_{N+k}(S^N;\ p)$ the p-primary component of the $(N+k)$-th homotopy group $\pi_{N+k}(S^N)$ of N-sphere S^N. In this paper, N denotes always a sufficiently large integer, in particular $N>k+1$ for the group $\pi_{N+k}(S^N)$ which does not depend on $N(>k+1)$ and is the k-th stable homotopy group $\pi_k(\mathfrak{S})$ of the sphere.

For $k<2p^2(p-1)-3$, the stable groups $\pi_{N+k}(S^N;\ p)$ are determined and stated as follows (p: *an odd prime*):

(A) $\pi_{N+2rp(p-1)-1}(S^N;\ p)=Z_{p^2}$ *for* $1\leqq r\leqq p-2$ *and* $=Z_{p^2}+Z_p$ *for* $r=p-1$;

(B) $\pi_{N+k}(S^N;\ p)=Z_p$ *for the following values of* k:

$k=2t(p-1)-1$ *where* $1\leqq t<p^2$ *and* $t\not\equiv 0 \pmod p$,

$=2(rp+s)(p-1)-2(r-s)$ *where* $0\leqq s<r\leqq p-1$,

$=2p^2(p-1)-2p$,

$=2(rp+s+1)(p-1)-2(r-s)-1$ *where* $0\leqq s<r\leqq p-1$ *and* $r-s\neq p-1$;

(C) $\pi_{N+k}(S^N;\ p)=0$ *for the other values of* $k<2p^2(p-1)-3$.

For example, $\pi_{N+2p(p-1)-2}(S^N;\ p)=Z_p$ and $\pi_{N+2p(p-1)-1}(S^N;\ p)=Z_{p^2}$.

Methods emploied here are the same as in [4] by determining the $\mathscr{S}^*$-module structure of stable cohomology groups of Postnikov complexes K_k over the sphere. Several difficulties may occur, the first one is related closely with the values of the above example, and it is removed by the aid of the results in the preceeding paper [6]. The second difficulty occurs in the dimensions about