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An example of deformations of complex analytic bundles

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In a recent paper Y. Matsushima studied the family of complex analytic bundles over a complex torus T which have $GL(r, C)$ as the structural group and which have holomorphic connections (see [5]). Particularly definitive are the results that if $r=2$, then those bundles which have holomorphic connections are those which are associated to the universal covering space of T, with respect to various representations of its fundamental group into $GL(2, C)$, and that the set of the indecomposable $GL(2, C)$-bundles over T with holomorphic connections are in one-to-one correspondence, in a natural way, with the product of the Picard variety $\mathfrak{P}$ of T and a complex projective space P of dimension one less than that of T. (See theorems 4 and 5 in [5].)

On the other hand it is clear that the decomposable $GL(2, C)$-bundles with holomorphic connections over a compact Kähler manifold M are in one-to-one correspondence with the points of the symmetric product V of the Picard variety $\mathfrak{P}$ of M with itself. We shall construct a non-singular variety W by a monoidal transformation applied to V. It will be shown that W contains a submanifold X homeomorphic to $\mathfrak{P}\times P$, that there exists an analytic $GL(2, C)$-bundle E over $M\times W$ which gives rise to a family of $GL(2, C)$-bundles over M parametrized by W, each of its member having holomorphic connections, and that $w\in X$ gives an indecomposable bundle.

Thus we see the true nature of Matsushima's theorem 5 and, at the same time, we see that the indecomposable bundles can be considered as limits of decomposable ones. Furthermore, if we compare this family with that of decomposable bundles parametrized