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Contributions to Riemann-Roch's theorem

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Introduction.

In the present paper we shall give at first a functiontheoretic new proof of the well-known Riemann-Roch's theorem for closed Riemann surfaces. Main ideas lie in the making use of Riemann's periods relation and the theory of linear spaces, that is, by considering some vector spaces consisting of Abelian differentials, linear functionals over them and dual spaces (vector spaces of linear functionals) we get two converse inequalities, therefore the equality, which implies our conclusion. From this point of view the relations between these spaces will be clarified.

In the second paragraph we treat, under the following restrictions, the case of non compact Riemann surfaces by the same method and we obtain an extension of R. Nevanlinna's theorem (Th. 2. 3) which is valid for square integrable differentials on Riemann surfaces $\in O_G$ of finite genus, where O_G denotes the class of Riemann surfaces which do not possess a Green's function. Here our restrictions are as follows;

(i) The basic Riemann surface (of finite or infinite genus) R belongs to O_{HD}, i.e. the class of Riemann surfaces admitting no non-vanishing total harmonic differential which is square integrable on R.

(ii) The Abelian differentials should be square integrable on R except the neighborhoods of a finite number of possible singularities. It is known that the inclusion relation $O_G \subset O_{HD}$ is proper only if the genus of R is infinite[1].

Finally, as an application of our theorem, a representation of

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1) Ahlfors and Royden [3] or Tôki [15].