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On the theory of Henselian rings, III.*

By

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In the paper [1], we defined the notion of Henselizations of normal quasi-local rings and proved generalized Hensel lemma in Henselian valuation rings, and in the paper [2] we proved the properties of Henselizations of normal quasi-local rings and of quasi-local integral domains.

In the present paper, we shall define the Henselization of an arbitrary quasi-local rings and we shall prove that if a Henselian ring $\mathfrak{h}$ dominates a quasi-local ring $\mathfrak{o}$ then there exists one and only one $\mathfrak{o}$-homomorphism from the Henselization of $\mathfrak{o}$ into $\mathfrak{h}$. Besides some other properties of Henselizations, we shall discuss unramifiedness. On the other hand, since the paper [2] contains some errors, corrections to the paper will be given in §1.

§1. Corrections to the paper [2].

(1) In §4 ([2, Chap. II]), we stated 4 lemmes (Lemmas 4-7). Among them, Lemma 6 is not correct (the others are correct).

What we should prove in §4 are really as follows:

Let $\mathfrak{o}$ be a normal quasi-local ring with maximal ideal $\mathfrak{p}$ and let $\mathfrak{q}$ be a prime ideal of $\mathfrak{o}$. Let $\bar{\mathfrak{o}}$ be an almost finite separable normal extension of $\mathfrak{o}$ with Galois group G and let $\bar{\mathfrak{p}}$ be a maximal ideal of $\bar{\mathfrak{o}}$. Let $\tilde{\mathfrak{o}}$ be the decomposition ring of $\bar{\mathfrak{p}}$ and set $\tilde{\mathfrak{p}}=\bar{\mathfrak{p}}\cap\tilde{\mathfrak{o}}$, $\mathfrak{o}^*=\tilde{\mathfrak{o}}_{\tilde{\mathfrak{p}}}$. We denote by $\mathfrak{q}^*$ and S an arbitrary prime divisor of $\mathfrak{q}\mathfrak{o}^*$ and the complement of $\mathfrak{q}$ in $\mathfrak{o}$. Then, (i) *$\mathfrak{q}^*\cap\mathfrak{o}=\mathfrak{q}$,* (ii) *$\mathfrak{q}\mathfrak{o}^*_{\mathfrak{q}^*}=\mathfrak{q}^*\mathfrak{o}^*_{\mathfrak{q}^*}$,* (iii) *$\mathfrak{o}^*_S/\mathfrak{q}\mathfrak{o}^*_S$ is Noetherian and* (iv) *$\mathfrak{q}\mathfrak{o}^*$ is the intersection of all the $\mathfrak{q}^*$.*

(i) was proved in Lemma 4 (in a more general form) and the

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