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Note on coefficient fields of complete local rings*

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Let R and R' be complete local rings (which may not be Noetherian) such that R' is integral over R. Then it is known that if the residue class field of R' is separable over that of R then any coefficient ring of R is extendable to that of R'.

The purpose of the present paper is to prove the following

Theorem. *Assume that R contains a field of characteristic* $p \neq 0$ *and that* $R'^p \subseteq R$. *Then there is a coefficient field of R which is extendable to that of* R'.

We shall give later some remarks on the case where $R'^{p^n} \subseteq R$.

The following fact was proved by Cohen[1]:

Existence lemma. *Let* R' *be a complete local ring, with residue class field* K', *containing a field of characteristic* $p \neq 0$. *Let* $\{\bar{c}_\sigma\}$ *be a p-basis of* K' *and let* $\{c_\sigma\}$ *be a set of representatives of* $\{\bar{c}_\sigma\}$ *in* R'. *Then there exists a coefficient field of R which contains all the* c_σ.

Using this existence lemma, we shall prove the theorem. Let K and K' be the residue class fields of R and R' respectively. Since $R'^p \subseteq R$, we have $K'^p \subseteq K$. Let $\{\bar{c}_\sigma\}$ be a maximal subset of K among those which are p-independent over K'^p and let $\{\bar{c}'_{\sigma'}\}$ be such that $\{\bar{c}_\sigma, \bar{c}'_{\sigma'}\}$ forms a p-basis of K'. Then it is obvious that K is generated by the $\bar{c}_\sigma$ over K'^p. Let k' be a coefficient field of R' containing representatives c_σ of $\bar{c}_\sigma$ in R. Then $k'^p(\{c_\sigma\})$ is a coefficient field of R contained in k,' which proves the theorem.

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1) I. S. Cohen, On the structure and ideal theory of complete local rings, Trans. Amer. Math. Soc. vol. 59 (1946), pp. 54-106.