MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXXII, Mathematics No. 1, 1959.

Note on a chain condition for prime ideals*

By

Masayoshi NAGATA

(Received Dec. 27, 1958)

We say that a ring R is of *finitely generated type* over a ring S if R is a ring of quotients of a finitely generated ring over S.

We say that the *dimension formula* holds for a local integral domain S[1] if the following formula is true for any local integral domain R which dominates S and which is of finitely generated type over S:[2]

$$\text{rank } R+\dim_{S/\mathfrak{n}} R/\mathfrak{m} = \text{rank } S+\dim_{((S))}((R)),$$

where $\mathfrak{n}$, $((S))$; $\mathfrak{m}$, $((R))$ denote the maximal ideals and the fields of quotients of S and R respectively.

On the other hand, we introduced in [C. P.][3] the *second chain condition* for prime ideals, which is stated as follows if we restrict ourselves only to integral domains:

The first chain condition holds in an integral domain R if and only if every maximal chain of prime ideals in R has length equal to rank R. The second chain condition holds in an integral domain R if and only if the first chain condition holds in any integral extension[4] of R.

It should be remarked here that if R is a Noetherian integral domain, the second chain condition for R is equivalent to each of the following conditions, as was shown in [C. P.]:

Condition C′: The first chain condition holds in every *finite*

* The work was supported by a research grant of National Science Foundation.

1) The same notion can be defined for general local rings, but is a trivial generalization.

2) In general, if S is Noetherian, then we have the inequality rank $R+\dim R/\mathfrak{m} \leq \text{rank } S+\dim((R))$.

3) We refer by [C. P.] the paper "On the chain problem of prime ideals" Nagoya Math. J. 10 (1956).

4) An integral extension of an integral domain R is an integral domain which is integral over R.