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Proof that any birational class of non-singular surfaces satisfies the descending chain condition.

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In our monograph "Introduction to the problem of minimal models in the theory of algebraic surfaces" (Publications of the Mathematical Society of Japan, no. 4; this monograph will be referred to as IMM) we have stated the proposition that *each birational class of non-singular varieties satisfies the descending chain condition* (see IMM, Proposition III. 1.3, p. 79), it being understood that the underlying partial ordering of the class is the one in which $V<V'$ if V' dominates V. In the quoted monograph we gave a proof based on the theorem of Neron-Severi. We have also mentioned the existence, in the case of surfaces, of a sheaf-theoretic proof due to Serre (a similar sheaf-theoretic proof has been given recently by Matsumura in an unpublished paper). Finally we have alluded in IMM to a forthcoming note in Mem. Col. Sci. of Kyoto University in which we proposed to prove the above descending chain condition for algebraic surfaces by elementary algebro-geometric considerations, using properties of *exceptional cycles* and the *anticanonical system* $|-K|$. This is the note in which we propose to give this proof.

§ 1. Exceptional cycles of the first kind.

Let F be a non-singular surface (over an algebraically closed

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