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On a characterization of a Jacobian variety[1),2)]

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To Prof. O. ZARISKI

Let J^g be a Jacobian variety of a complete non-singular curve Γ, φ be a canonical mapping of Γ into J and let Θ be a canonical divisor corresponding to $\varphi(\Gamma)$. It is known, in the classical case, that $\Theta_{u_1} \cdots \Theta_{u_{g-1}}$ is numerically equivalent to $(g-1)!\ \varphi(\Gamma)$ and the self-intersection number of Θ is $g!$. Originally these are due to Poincaré and later Castelnuovo gave an algebro-geometric proof for the first (cf. Castelnuovo [1]). Castelnuovo's idea is very simple but the proof depends upon a rather difficult result, the irreducibility of the variety of moduli of curves of the given genus. First, we shall prove them by using the theorem of Riemann-Roch and an equivalence criterion for numerical equivalence we shall discuss. Later in the Appendix, we shall prove them using Weil's idea, which was communicated to the writer by him. Next let A be an Abelian variety of dimension n, X be an irreducible subvariety of dimension $n-1$ on A such that the self-intersection number of X is $n!$ and that $X_{u_1} \cdots X_{u_{n-1}}$ is numerically equivalent to $(n-1)!\,C$, where C is a positive 1-cycle on A. Then we shall show that C is irreducible, non-singular, A is the Jacobian variety of C, C is canonically embedded into A and that X is a canonical divisor corresponding to C. Therefore, we can say that the two numerical relations, together with the irreducibility of the divisor, characterizes a canonically polarized Jacobian variety completely. In §1, we define an endomorphism $\alpha(X, Y)$ relative to a pair (X, Y) of

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2) We shall follow the terminology and conventions of Weil [6], [8].