MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXXII, Mathematics No. 2, 1959.

The Frobenius theorem and the duality theorem on an Abelian variety[1)]

By

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Received August 31, 1959

(Communicated by Prof. Akizuki)

The main purpose of this note is to establish the following theorem on an abstract abelian variety:

Let A be an abelian variety of dimension n, and let X be a divisor on it; then the degree $\nu(\varphi_X)$ of the homomorphism φ_X of A into its dual $\hat{A}$[2)] is equal to $[(X^{(n)})/n!]^2$, where $(X^{(n)})$ means the n-fold intersection number of X.

If X is positive and non-degenerate, then the dimension $l(X)$ of the complete linear system $|X|$ is given by $(X^{(n)})/n!$ (cf. Nishi [6], Th. 3). Therefore, our theorem extends the classical Frobenius Theorem. The method used in this note is purely algebraic and is valid not only for the classical case but also for the modular case. The so-called Duality Theorem "the double dual $\hat{\hat{A}}$ of A is isomorphic to A" can be obtained as a simple corollary of the Frobenius Theorem.

I have received kind advices from Matsusaka and also from Nakai to whom I wish to express here my hearty thanks.

§ 1. Preliminaries.

Let A^n be an abelian variety and let X be a divisor on it; the set $\mathfrak{G}_X$ of all points t of A such that $X_t \sim X$ is a subgroup of A. By $\hat{A}$ we shall denote the dual of A (i.e. the Picard variety of A). Then it is well known that the two abelian varieties A and $\hat{A}$ are isogenous. The mapping $\varphi_X : u \rightarrow \hat{u}$, where $\hat{u}$ is the

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1) We shall use freely the notations and the results in Weil [9]. Numbers in brackets refer to the bibliography at the end.

2) See the definitions in § 1.