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p-primary components of homotopy groups

IV. Compositions and toric constructions

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Here we shall use the following notations. $G_k=\pi_k(\mathfrak{S})$ is the k-th stable homotopy group of the sphere and $\pi_k(\mathfrak{S};p)$ is its p-primary component. $\alpha\circ\beta\in G_{h+k}$ indicates the composition of $\alpha\in G_h$ and $\beta\in G_k$. The toric construction

$$\{\alpha, \beta, \gamma\}\in G_{h+k+l+1}/(\alpha\circ G_{k+l+1}+\gamma\circ G_{h+k+1})$$

will be defined if $\alpha\in G_h$, $\beta\in G_k$ and $\gamma\in G_l$ satisfy the condition $\alpha\circ\beta=\beta\circ\gamma=0$. This is different only in sign to one given in Chapter 5 of [4]. Denote by $\alpha_1\in G_{2p-3}$ a generator of $\pi_{2p-3}(\mathfrak{S};p)=Z_p$ and choose elements α_t of $G_{2t(p-1)-1}$ inductively such that $\alpha_t\in\{\alpha_{t-1}, p\iota, \alpha_1\}$. Denote by β_s, $1\leqq s<p$, a generator of $\pi_{2(sp+s-1)(p-1)-2}(\mathfrak{S};p)=Z_p$, and denote by β_1^r the r-fold iterated composition $\beta_1\circ\cdots\circ\beta_1$ of β_1. There exist elements α'_{rp}, $1\leqq r<p$, such that $p\alpha'_{rp}=\alpha_{rp}$ for $1\leqq r<p-1$ and $p\alpha'_{(p-1)p}=\alpha_{(p-1)p}+x\alpha_1\circ\beta_1^{p-1}$ for some integer x. Then these elements and their compositions generate the p-components $\pi_k(\mathfrak{S};p)$ of the stable groups G_k for $k\leqq 2p^2(p-1)-3$.

Theorem 4.15. (*cf.* Theorem 3.13 of [6]).

$$\pi_{2rp(p-1)-1}(\mathfrak{S};p)=Z_{p^2}=\{\alpha'_{rp}\}\quad for\ 1\leqq r<p-1,$$
$$=Z_{p^2}+Z_p=\{\alpha'_{(p-1)p}\}+\{\alpha_1\circ\beta_1^{p-1}\}\quad for\ r=p-1,$$
$$\pi_{2t(p-1)-1}(\mathfrak{S};p)=Z_p=\{\alpha_t\}\quad for\ 1\leqq t<p^2\ and\ t\not\equiv 0\ (\mathrm{mod}\ p),$$
$$\pi_{2(rp+s)(p-1)-2(r-s)}(\mathfrak{S};p)=Z_p=\{\beta_1^{r-s-1}\circ\beta_{s+1}\}\quad for\ 0\leqq s<r\leqq p-1,$$
$$\pi_{2(rp+s+1)(p-1)-2(r-s)-1}(\mathfrak{S};p)=Z_p=\{\alpha_1\circ\beta_1^{r-s-1}\circ\beta_{s+1}\}$$
$$for\ 0\leqq s<r\leqq p-1\ and\ r-s\neq p-1,$$