Memoirs of the College of Science, University of Kyoto, Series A
Vol. XXXII, Mathematics No. 2, 1959.

On the strong stability and boundedness of solutions of ordinary differential equations

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(Received September 8, 1959)

In recent years many authors[1] have studied the problem of determining gauge functions, that is, Lyapunov functions for various types of stability and boundedness of solutions of ordinary differential equations. In this paper we shall show that the function $D(P, Q)$, introduced by H. Okamura[2] in connection with the uniqueness problem in the theory of ordinary differential equations (cf. Definition 1), will work as the above mentioned gauge function.

In §1 we shall define the Okamura function $D(P, Q)$. In §2 we shall obtain a necessary and sufficient condition for the trivial solution $x=0$ of the differential equation (1) to be strongly stable[3] in terms of the Okamura function. It should be noted that the Okamura function can be determined concretely by the given differential equation itself, though it may not be easy. In §3 we shall prove a regularization theorem which will connect our condition in §2 with that of well-known form. In §4 we shall discuss the strong boundedness problem by the same idea as for the strong stability in §2.

1. Okamura function.

In §1, §2 and §3 we consider the differential equation

$$(1) \qquad \frac{dx}{dt} = f(t, x)$$

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1) cf., for instant, Antosiewicz [1].
2) cf. Okamura [6], [7] and [8].
3) cf. Okamura [9] and cf. Yoshizawa [10].