MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXXII, Mathematics No. 2, 1959.

Determination of the second fundamental form of a hypersurface by its mean curvature

By

Makoto MATSUMOTO

(Received September 2, 1959)

It is well known [1, p. 200], [4, p. 188] that, if a hypersurface of an euclidean space is of type more than two, then the second fundamental form (II) is uniquely determined by its first fundamental form (I). On the other hand, in 1945, T. Y. THOMAS [5] shaw that the form (II) of a surface is determined in general by its form (I) and the mean curvature M. Therefore the imbedding of a 2-dimensional Riemannian space, which is assumed to be of type two, is uniquely determined by giving the mean curvature M, within rigid motions.

These results lead us to consider the imbedding of an $n(>2)$-dimensional Riemannian space of type two by giving the mean curvature M. Thus our problem is to *find the expression of the form* (*II*) *in terms of the form* (*I*), *the scalar M, and their derivatives.* The methods, by means of which Thomas deduced the expression of the form (II) of a surface, are not applicable to a hypersurface of general dimensional number, because he did not use the process of tensor-calculus, and further the simple equations (1.1) giving the curvature tensor of a surface do not hold good for a hypersurface, except when the hypersurface is of constant curvature.

In the first part of this paper the problem of Thomas [5] is treated by the process of tensor-calculus. We shall show that the determination of the form (II) will be done by solving a system of *linear* equations (1.13).

The second part of this paper is devoted to generalize the problem to the case of dimensions $n>2$. The expressions of the covariant derivatives of the second fundamental tensor H_{ij} are also obtained, but, in this time, their symmetry leads us to some