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Theory of Abelian integrals and its applications to conformal mappings

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Introduction.

The main aim of the present paper is to develop the theory of Abelian integrals on an arbitrary open Riemann surface R. For this purpose we shall introduce in sec. 5 the notion of canonical potentials on R which is a generalization of the normalized potentials. Roughly speaking, a normalized potential takes the constant value zero on the ideal boundary of R, while a canonical one is characterized by the fact that it takes respective real constant value on each ideal boundary component, and canonical differentials are defined as meromorphic differentials derived from canonical potentials. However, the one that attracts our interest particularly is the class $\mathfrak{R}$ of the semi-exact canonical differentials (or integrals of these) which have, by definition, no periods along dividing cycles. Then we are able to establish theorems of Riemann-Roch and Abel on R in terms of elements of $\mathfrak{R}$ which have the analogous formulations as in classical theories. Further finding that the functions of $\mathfrak{R}$ possess an extremal property, we know that our theory have close connections with canonical conformal mappings. Now we show in the following the brief program of this paper.

§ I contains some notes on harmonic measures which are simple fundamental canonical potentials. In § II the definition of canonical differentials is given along with some of their properties. Above all, the uniqueness theorem (Lemma 5) will be powerful for later use. Next, the existence of three kinds of elementary differentials in $\mathfrak{R}$ is proved by using the theory of orthogonal decomposition due to Nevanlinna-Virtanen. Another treatment of this existence theorem will be given in § IV.