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On some ergodic transformations in metric spaces

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Introduction. Let $(\Omega, \mathfrak{S}, \mu)$ be a measure space and T an invertible measure-preserving transformation on Ω. Let $\mathfrak{B}(\Omega, \mathfrak{S}, \mu)$ be the set of all equivalence classes of measurable sets of Ω, where two measurable sets E and F are called equivalent if and only if their symmetric difference $E \ominus F$ has measure 0. Then the set $\mathfrak{B}(\Omega, \mathfrak{S}, \mu)$ is a Boolean algebra under the natural Boolean operations and the measure μ can be considered as a measure on this Boolean algebra. (This Boolean algebra $\mathfrak{B}(\Omega, \mathfrak{S}, \mu)$ is called the measure algebra associated with the measure space $(\Omega, \mathfrak{S}, \mu)$.) The transformation T induces in a natural way a measure-preserving automorphism of the measure algebra $\mathfrak{B}(\Omega, \mathfrak{S}, \mu)$. The set of all complex valued μ-measurable functions $f(x)$ for which $\int_\Omega |f(x)|^2 d\mu(x) < \infty$ forms a Hilbert space $L_2(\Omega, \mathfrak{S}, \mu)$, and the transformation T induces a unitary operator on $L_2(\Omega, \mathfrak{S}, \mu)$ if we correspond to every $f(x) \in L_2(\Omega, \mathfrak{S}, \mu)$ a function $g(x)$ such that $g(x) = f(T(x))$. In this way an invertible measure-preserving transformation T on Ω can be regarded as a measure-preserving automorphism of $\mathfrak{B}(\Omega, \mathfrak{S}, \mu)$ or a unitary operator on $L_2(\Omega, \mathfrak{S}, \mu)$.

Let us suppose that S and T are invertible measure-preserving transformations on Ω. To discuss relations between such two transformations there has been introduced the concepts "similarity", "conjugacy" and "equivalence". First, S and T will be called (geometrically) similar if there exists an invertible measure-preserving transformation Q on Ω such that $S = Q^{-1}TQ$. (In this case we say that two transformations S and T are essentially the