MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXXIII, Mathematics No. 2, 1960.

On the theory of regular functions in banach algebras

By

Yoshimichi MIBU

(Received Sep. 3, 1960)

(Communicated by Prof. A. Komatu)

Introduction. The present paper is concerned with the general problem of extending the classical theory of regular functions of a complex variable. This problem was discussed by many authors from various directions. Our approach differs from most of the others in two main respects, namely, in the type of domain and range of the functions and in the definition of regularity. We deal with functions which have for their domains and ranges subsets of a commutative Banach algebra with unit element and we use a definition of regularity introduced by E. R. Lorch [1]. It is known [4] that a regular function by this definition is differentiable in the Fréchet sense but not every Fréchet-differentiable function on a commutative Banach algebra is regular in the Lorch sense. Accordingly, the Lorch theory is the richer.

For the most part, the development of the Lorch theory goes parallel with that of the classical theory. As one would expect, the Cauchy integral theorem and formula occupy a central position and yield the Taylor expansion. Our purpose is to discuss his theory in detail and to get more precise consequences. Our investigations contain some results which were not studied by Lorch. For example we have discussed the functions $\log z$ and $\sqrt[m]{z}$ from the view point of analytic continuations. The logarithmic function was also introduced by Lorch, but his definition seems to be artificial.

The main results of this paper is the theory of analytic continuations in which Theorem 3.1 employ an essential role. And