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Existence of a bounded solution and existence of a periodic solution of the differential equation of the second order

By

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1. Introduction. We consider a differential equation of the second order

$$x'' = F(t, x, x'), \tag{1}$$

where $F(t, x, x')$ is periodic of t. Massera has proved that if all the solutions exist in the future and if one of them is bounded in the future, then a periodic solution exists [1]. Therefore, even if all the solutions are not bounded, when we see the existence of a bounded solution, we can prove the existence of a periodic solution in some cases.

In this paper we discuss the existence of a bounded solution and we apply it to the existence of a periodic solution.

Now we assume that $F(t, x, x')$ is continuous in $I \times R^1_x \times R^1_y$, where I is the interval $0 \leqq t < \infty$ and R^n is the n-dimensional Euclidean space. For Theorem 1, the periodicity of $F(t, x, x')$ is not necessary.

2. Existence of a bounded solution. We shall obtain an existence theorem of a bounded solution by considering the boundary value problem.

Theorem 1. *Suppose that two functions* $\overline{\omega}(t)$ *and* $\underline{\omega}(t)$ *are*

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