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On the homotopy-commutativity of groups and loop spaces

By

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Introduction. It is well known that the loop space $\Omega(B)$ in B is homotopy-commutative if B is an H-space. Furthermore, it follows that the group G, which is also a CW-complex, is homotopy-commutative if its classifying space B_G is an H-space, because there is an H-homomorphism $f: G \to \Omega(B_G)$ which is also a weak homotopy equivalence (cf. [9], Theorem 1 and also [12], Theorem 2). It is our purpose of this paper to study about the inverses of these facts.

In the first part, the notion of the strong homotopy-commutativity is considered, and it is proved that the strong homotopy-commutativity of $\Omega(B)$ or G and being B or B_G an H-space are equivalent (Theorems 4.2 and 4.3).

In the second part, an exact sequence of the sets of homotopy classes for a fibre space with certain conditions are considered (Theorem 6.5), and the image of the map $\pi(X, Y) \to \pi(\Omega X, \Omega Y)$ is studied (Lemma 7.4). Finally, it is proved that only the homotopy-commutativity of $\Omega(B)$ or G is equivalent to being B or B_G an H-space for certain kinds of spaces, (Theorems 8.1 and 8.2).

Part I. Strong homotopy-commutativities

1. Commutative groups. Let G be a countable CW-group, and $p: E \to B$ an universal bundle with group G where a classifying space B is a countable CW-complex.[1] If G is *commutative*, then the map

1) A group G is called a countable CW-group if G is a countable CW-complex such that the map $g \to g^{-1}$ of $G \to G$ and the multiplication $G \times G \to G$ are both cellular maps. Milnor, [3], Theorem 5.1.(1), proved that such a group has a countable CW-complex as a classifying space.