

On P. J. Myrberg's approximation theorem on Fuchsian groups

By

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1. Introduction.

P. J. Myrberg [3] proved an approximation theorem for Fuchsian groups having fundamental domain of finite non-euclidean area. The purpose of the present paper is to extend it for Fuchsian groups of divergence type and, further, show that the validity of his approximation theorem implies conversely the divergence type of Fuchsian groups. As is known ([10]), a Fuchsian group is of divergence type if and only if the corresponding Riemann surface is of class O_G . Finally in 5 and 6 we shall state some related results and problems.

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2. Main Theorem.

We begin with stating our main result. Let G be a Fuchsian group of linear transformations:

$$S_n: z' = e^{i\alpha_n} \cdot \frac{z - a_n}{1 - \bar{a}_n z} \quad (|a_n| < 1) \quad (n = 0, 1, 2, \dots),$$

which leave $|z| < 1$ invariant and let D_0 be its normal fundamental domain which contains $z=0$. The quantity

$$\sigma(D_0) = 4 \iint_{D_0} \frac{r dr d\theta}{(1-r^2)^2}, \quad z = re^{i\theta},$$

is called its *non-euclidean area*. The boundary of D_0 consists of