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A note on the pseudo-compactness of the product of two spaces

By

Hisahiro TAMANO

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As is well known, the Stone-Čech compactification of a product space is not generally identical (more precisely, homeomorphic) with the product of the Stone-Čech compactifications of coordinate spaces. M. Henriksen and J. R. Isbell [5] pointed out that the relation $\beta(X\times Y)=\beta X\times\beta Y$[1] implies the pseudo-compactness of the product $X\times Y$[2,3]. Recently, the converse has been established by I. Glicksberg [4]. He proved more generally that the relation $\beta(\Pi X_\alpha)=\Pi\beta X_\alpha$ holds true if and only if ΠX_α[4] is pseudo-compact.

In this note, we shall restrict ourselves to consider the product of two spaces, and give some conditions equivalent to that the relation $\beta(X\times Y)=\beta X\times\beta Y$ hold. We shall show that $\beta(X\times Y)=\beta X\times\beta Y$ if and only if the tensor product $C^*(X)\otimes C^*(Y)$ is dense in $C^*(X\times Y)$.

The pseudo-compactness of the product $X\times Y$ implies the pseudo-compactness of each coordinate space. However, it is not true that the product of pseudo-compact spaces must be pseudo-compact[5]. Several additional conditions sufficient to insure the pseudo-compactness of the product of pseudo-compact spaces are given and discussed in [1], [4] and [5]. We shall generalize those results in somewhat unific form.

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1) Throughout, we shall consider X as a subspace of βX.

2) The trivial case that X or Y is a finite set will be excluded throughout. If X is a finite set, then $\beta(X\times Y)=\beta X\times\beta Y$ for any space Y.

3) T. Ishiwata [7] has proved that if $\beta(X\times X)=\beta X\times\beta X$, then X is totally bounded for any uniform structure of X. (X is pseudo-compact if and only if it is totally bounded for any uniform structure of X. C. f. *T. Ishiwata*: *On uniform spaces, Sugaku Kenkyuroku*, Vol. 2 (1953) (*in Japanese*).)

4) ΠX_α denotes the product of X_α.

5) C.f. [9], [10].