On the harmonic boundary of an open Riemann surface, II

By

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Introduction

The present paper contains some generalizations and supplements of our previous results [6]. Furthermore the relation between harmonic boundary points and the minimal functions (in $\underline{HD}$) will be studied.

We shall denote by R an open Riemann surface and by R^* the Royden compactification of R. In §1 the structure of R^* and some definitions are stated. §2 is concerned with the harmonic measure with respect to any compact subset of harmonic boundary Δ of R. In particular, the harmonic measure with respect to a single point becomes a minimal function in class $\underline{HD}$ studied by Constantinescu-Cornea [1] and one-to-one correspondence between minimal functions (in $\underline{HD}$) and some points in Δ will be established. These results are the contents of §3. Finally in §4 from our point of view we shall study the properties of non-compact subregions G on R, by which some theorems in previous paper [6] will be made more clear and complete. Particularly, Theorem 6 (or 6′) gives the characterization of $G \notin SO_{HD}$, which has some remarkable applications.

§1. Structure of R^*

1. Let R denotes an arbitrary open Riemann surface. For the sake of definiteness and convenience we shall state briefly the structure of the compactification R^* of R (for some proofs see Gelfand-Silov [2] and Royden [11]. cf. Nakai [10] for another approach). Let BD denotes the class of bounded continuous func-