MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXXIII, Mathematics No. 3, 1961.

On holomorphic families of fiber bundles over the Riemannian sphere[1]

By

Helmut RÖHRL

(Communicated by Prof. A. Komatu, February 28, 1961)

In a recent paper A. Grothendieck [11] classified the holomorphic fiber bundles over the Riemannian sphere P^1 whose structure group is reductive. This classification is based on the fact that each holomorphic vector bundle over P^1 splits into holomorphic line bundles, a result that is essentially due to G. D. Birkhoff [1]. In the present paper we raise the corresponding question for holomorphic families of fiber bundles $\mathcal{B} \to \mathcal{V}$ over a holomorphic family $\mathcal{V} \xrightarrow{\pi} M$ of Riemannian spheres, i.e. holomorphic fiber bundles whose base space is the total space of a holomorphic fiber bundle with fiber P^1. It turns out (Theorem 2.2) that the splitting theorem is locally still valid provided one avoids a 1-codimensional analytic subset A of the parameter space M of the family. The exceptional set A is empty if the restrictions of the vector bundle to any two fibers are isomorphic (Theorem 2. 4). The splitting theorem permits one to prove that the set of all points $t \in M$ fulfilling

$$\dim_C H^0(V_t, \Omega(B_t)) \geq j$$

for some integer j is an analytic subset of M (Theorem 2. 3). In addition one gets a counterpart to a theorem of K. Kodaira and D. C. Spencer ([14], Theorem 18. 1) stating that for any point $t_0 \in M$ there is a neighborhood U such that

$$H^0(\pi^{-1}(U), \Omega(\mathcal{B})) \to H^0(V_t, \Omega(B_t))$$

---

1) This research was supported by the U.S. Air Force through the Air Force Office of Scientific Research under contract No. Af 49 (638)-885.