MEMOIRS OF THE COLLEGE OF SCIENCE, UNIVERSITY OF KYOTO, SERIES A
Vol. XXXIII, Mathematics No. 3, 1961.

On the construction of two-dimensional diffusion processes satisfying Wentzell's boundary conditions and its application to boundary value problems

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(Received Jan. 15, 1961)

0. Introduction. Consider a Markov process on a regular domain $\bar{D}$ in R^2 whose generator is given as an elliptic differential operator

$$(0.1)\quad \begin{aligned} Au(x, y) &= A_{11}(x, y)\frac{\partial^2}{\partial x^2}u(x, y) + 2A_{12}(x, y)\frac{\partial^2}{\partial x \partial y}u(x, y) \\ &+ A_{22}(x, y)\frac{\partial^2}{\partial y^2}u(x, y) + B_1(x, y)\frac{\partial}{\partial x}u(x, y) + B_2(x, y)\frac{\partial}{\partial y}u(x, y), \\ &\qquad \text{for} \quad u(x, y) \in C^2, \end{aligned}$$

in D associated with the boundary condition

$$\begin{aligned} \delta(x, y) \lim_{(x', y') \to (x, y)} Au(x, y) &= \frac{\partial}{\partial \xi^2_{(x,y)}}u(x, y) + M(x, y)\frac{\partial}{\partial \xi^1_{(x,y)}}u(x, y) + V(x, y) \\ &\times \frac{\partial^2}{\partial \xi^1_{(x,y)} \partial \xi^1_{(x,y)}}u(x, y) + \int_{\partial D}\Big\{u(x+x', y+y') - u(x, y) - \frac{\partial}{\partial \xi^1_{(x,y)}}u(x, y) \\ &\times \xi^1_{(x,y)}(x', y')\Big\}\nu_{(x,y)}(dx'dy'), \quad \text{for} \quad u(x, y) \in C^2,\ (x, y) \in \partial D, \end{aligned}$$

where $\delta(x, y) \equiv 0$ or 1, $V(x, y)$ is non-negative and $\nu_{(x,y)}(\cdot)$ is a measure on ∂D satisfying $\nu_{(x,y)}(\partial D - U_{(x,y)}) < +\infty$ and $\int_{\partial D}(\xi^1_{(x,y)}(x', y'))^2 \nu_{(x,y)}(dx'dy') < +\infty$ for any neighbourhood $U_{(x,y)}$ of (x, y). $\{\xi^i_{(x,y)}(x', y'),\ i=1, 2\}$ is a C^2-function on $\bar{D}$ and is a local coordinate in $U_{(x,y)}$ satisfying: $\xi^2_{(x,y)}(x' y') = 0$ if and only if $(x', y') \in \partial D$ for $(x', y') \in U_{(x,y)}$, $\xi^2_{(x,y)}(x', y') + (\xi^1_{(x,y)}(x', y'))^2 > 0$ if and only if $(x', y') \neq (x, y)$. It is easily seen from A. D. Wentzell's results [24] that this boundary condition is of the most general type provided