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Parametrization of a family of bundles

By

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Let Γ be a compact Riemann surface of genus g, and let $\hat{\Gamma}$, Π be respectively the universal covering surface and the fundamental group of Γ. Denote by A the group of the affine transformations on $\boldsymbol{C}^1$. In this paper we consider the set of complex analytic bundles E over Γ, which are associated to the bundle $\hat{\Gamma} \to \Gamma$ by representations $\rho : \Pi \to A$.

We shall show that this set has a natural analytic structure except for a singular part, and forms an analytic family of bundles (§2). We also add some remarks on the singular part (§3).

Generally, we follow notations in papers of Kodaira and Spencer.

§1. Structure of complex analytic family of bundles

In reference [5], Kodaira and Spencer proved three theorems on the existence of structure of complex analytic family in a regular differentiable family of deformations of compact analytic manifolds. In this section we shall prove variants of these theorems for the case of a family of bundles.

Let G be a complex Lie group of matrices and let $\mathcal{P} \xrightarrow{p} \mathcal{V} \xrightarrow{\varpi} M$ be a differentiable family of principal G-bundles over the family of compact complex manifolds $\mathcal{V} \to M$. From the fundamental diagram of sheaves for this family, we obtain the commutative diagram

$$\begin{array}{ccc} (T_M)_t & \xrightarrow{\eta_t} & H^1(V_t, \Sigma_t) \\ \| & & \downarrow \kappa_t \\ (T_M)_t & \xrightarrow{\rho_t} & H^1(V_t, \Theta_t) \,. \end{array}$$

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