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On a compactification of an open Riemann surface and its application

By

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Introduction

In this paper, we shall give a compactification (denoted by $R^*_{\mathfrak{F}}$) of an open Riemann surface R ($\notin 0_G$) such that HB-functions on R are extended continuously onto $R^*_{\mathfrak{F}}$. Likely as the Royden's compactification [15], the ideal boundary $R^*_{\mathfrak{F}}-R$ has the compact part (denoted by $\Delta_{\mathfrak{F}}$) with an important role with respect to HB-functions. After H. L. Royden, we shall call it the harmonic boundary of R, and it will be remarked in § 4 as the hyper Stone space (cf. [13]). In § 1, the compactification will be carried out by means of some family consisting of bounded continuous function on R. In § 2, some properties of $\Delta_{\mathfrak{F}}$ will be studied. In § 3, we shall study the generalized harmonic measure on R in relation to subsets of the harmonic boundary $\Delta_{\mathfrak{F}}$, where the generalized harmonic measure ω is characterized as follows: 1) $\omega \in HBP$, 2) $0<\omega<1$ and 3) $\omega \wedge (1-\omega)=0$ (cf. [5]). We shall define the harmonic measure Ω_α with respect to a compact subset α of $\Delta_{\mathfrak{F}}$ by the same manner as did in [7] and we shall show that Ω_α is the generalized harmonic measure and conversely a generalized harmonic measure is the harmonic measure with respect to a compact set of $\Delta_{\mathfrak{F}}$. And further we shall define the outer harmonic measure with respect to any subset of $\Delta_{\mathfrak{F}}$. We shall see that the outer harmonic measure is the Caratheodory outer measure with respect to the subsets of $\Delta_{\mathfrak{F}}$. In § 4, we shall introduce the integral representation of an HB-function. With