J. Math. Kyoto Univ.
1-3 (1962) 425-455.

On decomposable mappings of manifolds

By

Yoshihiro SAITO

(Received April 19, 1962)

In the present paper we study some relations between the singularities of mappings and the decomposable mappings of manifolds. Throughout this paper by a smooth mapping (function) we understand a C^∞-mapping (C^∞-function); M^n denotes an orientable closend n-dimensional C^∞-manifold, and R^n the n-dimensional Euclidean space.

We shall now recall briefly the definitions of the *singularities* $S_r(f)$, $S_{r,r'}(f)$, $\cdots$ of a mapping $f: M^n \to R^p$, $n \geqq p$, [3], [5].

Let $S_r(f)$ denote the set of points of M^n at which f has rank $p-r$. Suppose that $S_r(f)$ is an m-dimensional submanifold of M^n. Then $S_{r,r'}(f)$ is defined to be the subset of $S_r(f)$ consisting of points at which the mapping f restridted on $S_r(f)$ has rank $m-r'$. By the similar way we define the singularities $S_{r,r',\cdots,r''}(f)$.

We shall give a condition under which $S_r(f)$ is a submanifold of M^n. Let $G(f)$ be the graph of f, and associate to each point p of M^n the tangent space of $G(f)$ at $(p, f(p))$. Then we have a mapping, denoted by $d_1 f$, of M^n to B_0, the space of n-planes in the tangent spaces of $M^n \times R^p$.

B_0 is a fibre bundle over $M^n \times R^p$ whose fibre is the Grassmann manifold G_n^p, the space of n-planes in R^{n+p}. Denote $B_1 = \bigcup_q F_r(q)$ where $F_r(q) = (p-r, \cdots, p-r, p, \cdots, p)$ is the Schubert variety in the fibre $G_n^p(q)$ over a point q of $M^n \times R^p$. Then B_1 is a submanifold of B_0. Since we have $S_r(f) = d_1 f^{-1}(B_1)$, it follows that if the mapping $d_1 f$ is t-regular (transverse regular) on B_1 then $S_r(f)$ is a regular submanifold of M^n.

Now we suppose that $d_1 f$ is t-regular on B_1. Let m be the