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On some relations between the harmonic measure and the Lévy measure for a certain class of Markov processes

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1. Introduction. Consider a Markov process $x(t)$ on a locally compact separable metric space S with right continuous path functions and, given an open set D, let τ_D be the first passage time for the complement of D. The main purpose of this paper is to establish the following relation

$$E_x(e^{-\lambda\tau_D};\ x(\tau_D)\in E) = \int_D \bar{g}_\lambda^D(x, dy)n(y, E)^{1)},$$

under some appropriate conditions where $\bar{g}_\lambda^D(x, \cdot)$ is the *Green measure* of the subprocess on D:

$$\bar{g}_\lambda^D(x, \cdot) = E_x\left(\int_0^{\tau_D} e^{-\lambda t}\chi_\cdot(x_t)dt\right)^{2)}$$

and $n(y, E)$ is *Lévy measure* of this process:

$$n(y,E)\Delta t \sim P_y(x(\Delta t)\in E) \qquad (t\downarrow 0).$$

This relation was first introduced by J. Elliott and W. Feller [4] for the Cauchy process on the line $(-\infty, \infty)$ and was used for the investigation of the symmetric stable processes [3], [8].

It is natural to conjecture that

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1) The suffix x of E_x, P_x, etc. refers to the starting point.

2) $\chi_E(x)$ is the characteristic function of set E.