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Mixed problem for some semi-linear wave equation

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1. Introduction

Let us consider the equation of the form:

$$\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}+f(u)\frac{\partial u}{\partial t}+g(u) = 0\,. \tag{1.1}$$

Our problem is the so-called initial-boundary problem to this equation. Namely, we want to show that, under some conditions on $f(u)$ and $g(u)$ which will be given later, there exists always a unique, genuine, global solution $u(x, t)$, $(x\geq 0,\ t\geq 0)$, for any couple of the initial $u(x, 0)\in C^2$, $u_t(x, 0)\in C^1$ $(x\geq 0)$ and the boundary $u(0, t)=\psi(t)\in C^2$ $(t\geq 0)$ data. Of course, we assume the compatibility conditions among these data up to order 2. Namely, $u(0, 0)=\psi(0)$, $u_t(0, 0)=\psi'(0)$ and

$$\psi''(0) = u_{xx}(0, 0)-f(u(0, 0))u_t(0, 0)-g(u(0, 0))\,.$$

Our first step is to obtain an a priori estimate of the solution u and its first derivatives in the maximum norm. It is evident that without any condition on the behaviors of f and g, we cannot expect to have such an estimate. We tried to make this condition less stringent. However, we remark here that from the first we restricted the type of equation to the form (1.1). We could extend our reasoning to another types of equations. However, we don't insist on this matter.

Our second step is to show the local existence theorem with respect to both the Cauchy data and the Goursat data. This